

QCD Sum Rule for $\Lambda(1405)$

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Abstract

Motivated by the recently constructed interpolation field for $S_{11}(1535)$, we propose a new interpolating field for $\Lambda(1405)$. Using this current, we calculate the mass of $\Lambda(1405)$ based on the conventional QCD sum rule analysis. By calculating the Wilson coefficients up to dimension 8 operators and taking into account the mass corrections from s-quark, we find the calculated mass of $\Lambda(1405)$ to be very close to its experimental value.

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For many years, QCD sum rule has been widely used to study the spectral properties of the hadrons since it was first introduced by Shifman, Vainshtein, and Zakharov [1]. The basic idea of the QCD sum rule is to describe the properties of a hadron, such as its mass or its coupling strength to other hadrons, in terms of QCD parameters. The starting point of the QCD sum rule is to introduce an appropriate interpolating field for the hadron of concern using the quark and the gluon fields. Successful description of the hadron depends partly on how strongly the interpolating field couples to the hadron. Therefore, the specific form of the interpolating field is usually determined by requiring it to have a sufficiently large overlap with the hadron's nonrelativistic quark wave functions as well as requiring the correct quantum numbers.

Most QCD sum rules so far focus on the lowest resonance while putting all other higher resonances into the continuum [2]. However, often in nuclear physics, it is important to understand the properties of the higher resonances and their decaying properties as they often participate in scattering processes. In this regards, as a starting point of studying the excited baryonic states, we have recently proposed a way to study the lowest excited state with negative-parity using the conventional QCD sum rule analysis [3]. There, the interpolating field for S_{11} is constructed by requiring it to have a large overlap with the nonrelativistic quark wave function of S_{11} , as suggested by the bag model [4] or the nonrelativistic quark model [5]. The reasonable prediction of the S_{11} mass and a stable Borel curve indicate that our interpolating field introduced there seems to overlap strongly with the S_{11} . In this work, we will apply the same method to study the Λ (1405). (In the following, we will denote Λ (1405) as Λ^* and Λ (1115) as Λ .)

First, we start by constructing the interpolating field for Λ . Λ has the quantum numbers of $I = 0$ and $S = -1$ composed of (u, d, s) quarks. The interpolating field for Λ in most QCD sum rule analysis is obtained from the *Ioffe current* of nucleon using SU(3) transformation. Here, we directly construct the Λ interpolating field following the procedure that has been used in constructing the nucleon interpolating field [6].

A simple way to construct the Λ interpolating field is to combine an up and a down quark to a spin 0 isoscalar diquark and then attaching an s-quark to the diquark. The quantum numbers of Λ are carried by the attached s-quark. So the possible forms of Λ interpolating field are

$$\eta_1 = \epsilon_{abc}(u_a^T C \gamma_5 d_b) s_c ; \quad \eta_2 = \epsilon_{abc}(u_a^T C d_b) \gamma_5 s_c . \quad (1)$$

In general, one can choose the interpolating field of Λ to be an arbitrary linear combination of these two fields,

$$\eta_\Lambda(t) = 2(t \eta_1 + \eta_2) . \quad (2)$$

This is one possible choice for the interpolating field with the known quantum numbers of Λ . Note that in the non-relativistic limit, η_1 reduces to three quarks in the s-wave state.

The interpolating field for Λ^* can be constructed by applying $z \cdot D$ to one of the quark fields above. The four vector z^μ is orthogonal to the momentum of Λ^* , q^μ ($z \cdot q = 0$) and $z^2 = -1$ so that, in the rest frame of Λ^* , $z \cdot D$ reduces to the derivative in the space direction. By introducing the covariant derivative in this way, the interpolating field will pick out one quark in the p-wave state and two quarks in the s-wave state, which is believed to be the quark configuration of Λ^* in the bag model [4] or nonrelativistic quark model [5]. From

the three possible choices of putting in the derivative, we choose the following interpolating field,

$$\eta_{\Lambda^*}(t) = 2\epsilon_{abc}[t (u_a^T C \gamma_5 z \cdot D d_b) s_c + (u_a^T C z \cdot D d_b) \gamma_5 s_c] . \quad (3)$$

This choice is preferred because the correlator calculated from this field has the nonzero contribution from the lowest order chiral breaking term, $\langle \bar{q}q \rangle$. One other choice with the covariant derivative acting on the u-quark yields the same expression for the correlator due the isospin symmetry. When the derivative acts on the s-quark, the correlator has zero contribution from $\langle \bar{q}q \rangle$. The parameter t will be determined by optimization procedure introduced in Ref. [3].

With this current, we consider the following time-ordered correlation function,

$$\Pi(q) = \int dx^4 e^{iq \cdot x} i \langle 0 | T[\eta_{\Lambda^*}(x) \bar{\eta}_{\Lambda^*}(0)] | 0 \rangle , \quad (4)$$

which will have two invariant scalar functions defined as,

$$\Pi(q) = \Pi_1(q^2, z^2) + \Pi_q(q^2, z^2) \not{q} . \quad (5)$$

The so called *theoretical side* of the sum rule is obtained by calculating the two scalar functions in the operator product expansion (OPE) whose Wilson coefficients are calculated using the fix-point gauge and the standard background-field techniques [7]. Our calculation up to dimension 8 operators gives

$$\begin{aligned} \Pi_q^{\text{ope}}(q) = & -\frac{q^6 \ln(-q^2)}{2^6 \times 3^2 \times 5 \times \pi^4} (t^2 + 1) - \frac{q^2 \ln(-q^2)}{2^9 \times 9 \times \pi^2} (9t^2 - 2t + 9) \langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \rangle \\ & - \frac{t^2 - 1}{12q^2} \langle \bar{q}q \rangle \langle g_s \bar{q} \sigma \cdot \mathcal{G} q \rangle - \frac{q^2 \ln(-q^2)}{60\pi^2} (t^2 + 1) m_s \langle \bar{q}q \rangle \\ & + \frac{(19t^2 + 4t + 19) \ln(-q^2)}{3^2 \times 2^6 \times 5 \times \pi^2} m_s \langle g_s \bar{q} \sigma \cdot \mathcal{G} q \rangle - \frac{t^2 + 1}{240} \frac{m_s}{q^2} \langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \rangle \langle \bar{q}q \rangle , \end{aligned} \quad (6)$$

$$\begin{aligned} \Pi_1^{\text{ope}}(q) = & \frac{q^4 \ln(-q^2)}{60\pi^2} (t^2 - 1) \langle \bar{q}q \rangle - \frac{q^2 \ln(-q^2)}{60\pi^2} (t^2 - 1) \langle g_s \bar{q} \sigma \cdot \mathcal{G} q \rangle \\ & + \frac{\ln(-q^2)}{120} (t^2 - 1) \langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \rangle \langle \bar{q}q \rangle - \frac{q^6 \ln(-q^2)}{2^7 \times 5 \times \pi^4} m_s (t^2 - 1) \\ & - \frac{q^2 \ln(-q^2)}{2^6 \times 3 \times \pi^2} (t^2 - 1) m_s \langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \rangle . \end{aligned} \quad (7)$$

Note that, in calculating these, we have made use of the properties, $z \cdot q = 0$ and $z^2 = -1$. Note also that we have calculated the mass correction (m_s) from s-quark up to dimension 8. The mass corrections come either from the s-quark propagator or from the short distant expansion of the condensate involving s-quark. As has been noted in Ref. [3], the introduction of the covariant derivative enhances the contribution from the dimension five operator, $\langle g_s \bar{q} \sigma \cdot \mathcal{G} q \rangle$, in Π_1^{ope} and constitutes the major part of the mass splitting between the positive-parity ground state baryon and the negative-parity first excited state. In our case, this condensate also contributes to Π_q^{ope} through s-quark mass. As in Ref. [3], we did not include the contribution from the dimension six operator, $\langle \mathcal{G}^3 \rangle$, as it is of order $\alpha^{3/2}$.

Due to the orthogonal property of z_μ ($z \cdot q = 0$), the *phenomenological side* of the correlator take the following form,

$$\Pi^{\text{phen}}(q) = -\left[\lambda_\Lambda^2 \frac{\not{q} - M_\Lambda}{q^2 - M_\Lambda^2 + i\epsilon} + \lambda_{\Lambda^*}^2 \frac{\not{q} + M_{\Lambda^*}}{q^2 - M_{\Lambda^*}^2 + i\epsilon}\right] + \text{continuum} \quad (8)$$

$$\equiv \Pi_1^{\text{phen}} + \not{q}\Pi_q^{\text{phen}}. \quad (9)$$

The lowest state with positive (negative) parity is identified with the Λ (Λ^*) where λ_Λ (λ_{Λ^*}) denotes its coupling strengths to our interpolating field. We have put all other resonances with higher masses into two separate continuum, one for the positive-parity resonances starting at s_+ , the other for the negative-parity resonances starting at s_- . The form of these continuum parts of the phenomenological side are obtained as follows. First, look at the corresponding continuum part of the imaginary part of the OPE. Second use dispersion relation to obtain the real part but assuming that the cut starts at the continuum threshold s_+ or s_- .

The parameter t is chosen in such a way that the resulting interpolating field does not couple to the positive-parity baryon while maximally couples to the negative-parity baryon. For this, we follow Ref. [3,8]. There we introduced the two scalar functions, Π_1^o and Π_q^o , at the rest frame ($\mathbf{q} = 0$), which are the two invariant functions in the “old-fashioned” correlation function $\Pi^o(q) = i \int d^4x e^{iqx} \theta(x_0) \langle 0 | \eta(x) \eta(0) | 0 \rangle$ [8]. In the sum (difference) of the two scalar functions, only the positive (negative) parity states contributes so that one can construct a finite energy sum rule which gives the coupling strength λ_Λ^2 as a function of t ,

$$\begin{aligned} 60\pi^2\lambda_\Lambda^2 = & \frac{t^2 + 1}{48\pi^2} \frac{(\sqrt{s_+})^8}{8} + \frac{5}{2^7 \times 3} (9t^2 - 2t + 9) \langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \rangle \frac{(\sqrt{s_+})^4}{4} \\ & + \frac{5\pi^2}{2} (t^2 - 1) \langle \bar{q}q \rangle \langle g_s \bar{q}\sigma \cdot \mathcal{G}q \rangle + (t^2 + 1) m_s \langle \bar{q}q \rangle \frac{(\sqrt{s_+})^4}{4} \\ & - \frac{19t^2 + 4t + 19}{48} m_s \langle g_s \bar{q}\sigma \cdot \mathcal{G}q \rangle \frac{(\sqrt{s_+})^2}{2} + \frac{\pi^2}{8} (t^2 + 1) m_s \langle \bar{q}q \rangle \langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \rangle \\ & + (t^2 - 1) \langle \bar{q}q \rangle \frac{(\sqrt{s_+})^5}{5} - (t^2 - 1) \langle g_s \bar{q}\sigma \cdot \mathcal{G}q \rangle \frac{(\sqrt{s_+})^3}{3} \\ & + \frac{\pi^2}{2} (t^2 - 1) \sqrt{s_+} \langle \bar{q}q \rangle \langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \rangle - \frac{3(t^2 - 1)(\sqrt{s_+})^7}{32\pi^2} \frac{7}{7} \\ & - \frac{5}{16} (t^2 - 1) m_s \langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \rangle \frac{(\sqrt{s_+})^3}{3}. \end{aligned} \quad (10)$$

The possible t is chosen so that λ_Λ^2 is zero. Normally there are two ts which give $\lambda_\Lambda^2 = 0$. We take t which yields larger value of $\lambda_{\Lambda^*}^2$.

Now we are in a position to determine the mass of Λ^* . We equate the phenomenological side of the two invariant functions in Eq. (9) to their corresponding OPE sides in Eq. (6) and Eq. (7). After taking the Borel transformation and taking the ratio of the two sum rules, we obtain a Borel sum rule for the M_{Λ^*} which yields,

$$\begin{aligned} M_{\Lambda^*} = & \left[-[\Delta_-^6 + \Delta_+^6] (t^2 - 1) \langle \bar{q}q \rangle + \frac{\Delta_-^4 + \Delta_+^4}{2} (t^2 - 1) \langle g_s \bar{q}\sigma \cdot \mathcal{G}q \rangle \right. \\ & \left. - \frac{\Delta_-^2 + \Delta_+^2}{4} \pi^2 (t^2 - 1) \langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \rangle \langle \bar{q}q \rangle + \frac{\Delta_-^8 + \Delta_+^8}{32\pi^2} (t^2 - 1) 9m_s \right] \end{aligned}$$

$$\begin{aligned}
& + \frac{\Delta_-^4 + \Delta_+^4}{32} 5(t^2 - 1) m_s \langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \rangle \Big] \\
& \times \left[\frac{\Delta_-^8 + \Delta_+^8}{16\pi^2} (t^2 + 1) + \frac{\Delta_-^4 + \Delta_+^4}{2^8 \times 3} \langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \rangle 5(9t^2 - 2t + 9) \right. \\
& + 5\pi^2 (t^2 - 1) \langle \bar{q}q \rangle \langle g_s \bar{q} \sigma \cdot \mathcal{G} q \rangle + \frac{\Delta_-^4 + \Delta_+^4}{2} m_s \langle \bar{q}q \rangle (t^2 + 1) \\
& - \frac{\Delta_-^2 + \Delta_+^2}{96} m_s \langle g_s \bar{q} \sigma \cdot \mathcal{G} q \rangle (19t^2 + 4t + 19) \\
& \left. + \frac{\pi^2}{4} m_s \langle \bar{q}q \rangle \langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \rangle (t^2 + 1) \right]^{-1}, \tag{11}
\end{aligned}$$

where we have defined

$$\Delta_{\pm}^2 = M^2 (1 - e^{-s_{\pm}/M^2}), \tag{12}$$

$$\Delta_{\pm}^4 = M^4 \left[1 - \left(1 + \frac{s_{\pm}}{M^2} \right) e^{-s_{\pm}/M^2} \right], \tag{13}$$

$$\Delta_{\pm}^6 = M^6 \left[1 - \left(1 + \frac{s_{\pm}}{M^2} + \frac{s_{\pm}^2}{2M^4} \right) e^{-s_{\pm}/M^2} \right], \tag{14}$$

$$\Delta_{\pm}^8 = M^8 \left[1 - \left(\frac{s_{\pm}}{M^2} + \frac{s_{\pm}^2}{2M^4} + \frac{s_{\pm}^3}{6M^6} \right) e^{-s_{\pm}/M^2} \right]. \tag{15}$$

We take the following QCD parameters which are conventionally found in literature,

$$\langle \bar{q}q \rangle = -(0.23 \text{ GeV})^3; \quad \langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \rangle = (0.35 \text{ GeV})^4; \quad m_s = 150 \text{ MeV}. \tag{16}$$

However, our final result is not sensitive to these parameters as long as they are varied within known error bars. The continuum thresholds are chosen as the positions of the next higher resonances:

$$s_+ = (1.6)^2 \text{ GeV}^2; \quad s_- = (1.67)^2 \text{ GeV}^2. \tag{17}$$

The most important parameter for our prediction is the quark-gluon condensate which is usually expressed in terms of the quark condensate $\langle \bar{q}q \rangle$:

$$\langle g_s \bar{q} \sigma \cdot \mathcal{G} q \rangle \equiv 2\lambda_q^2 \langle \bar{q}q \rangle. \tag{18}$$

The value of λ_q^2 , which represents the average virtual momentum of vacuum quarks, is not well known. In our previous work on S_{11} [3], we have found that the best prediction on S_{11} was obtained for $\lambda_q^2 = 0.435 \text{ GeV}^2$. So we take this value.

With these QCD parameters, the optimal t is found to be -2.781 . Putting all these into Eq. (11), we obtain the Borel curve shown in Fig. 1. Clearly we have a Borel plateau as the Borel mass increases. Our prediction for Λ^* mass which is obtained from most stable Borel plateau is $M_{\Lambda^*} = 1.42 \text{ GeV}$. This agrees with its experimental value of 1.405 GeV within 2 %.

In summary, we proposed a new interpolating field for $\Lambda^*(1405)$ constructed to reproduce its quark configuration in the bag model or the nonrelativistic quark model. By calculating the Wilson coefficients up to dimension 8 operators in the conventional QCD sum rule approach, we found that our interpolating field yields the mass of Λ^* close to its known value of 1405 MeV within 2 %. Therefore, our interpolating field couples strongly to Λ^* and can be used to study its other spectral properties.

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FIGURES

FIG. 1. Our prediction for Λ^* mass versus Borel mass.

